Black Hole Evaporation and Compact Extra Dimensions

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We study the evaporation of black holes in space-times with extra dimensions of size L. We first obtain a description which interpolates between the expected behaviors of very large and very small black holes and then show that the luminosity is greatly damped when the horizon shrinks towards L from a larger value. Analogously, black holes born with an initial size smaller than L are almost stable. This effect is due to the dependence of both the Hawking temperature and the grey-body factor of a black hole on the dimensionality of space. Although the picture of what happens when the horizon becomes of size L is still incomplete, we argue that there occurs a (first order) phase transition, possibly signaled by an outburst of energy which leaves a quasi-stable remnant.

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I. INTRODUCTION

The current interest in the possibility that there exist large extra dimensions [1,2] beyond the four dimensions of our every day experience is based on two attractive features of this proposal. First of all the hierarchy problem is by-passed because the radiative stability of the weak scale in this proposal is due to the identification of the ultraviolet cutoff of the theory with the electroweak energy scale Λ_{EW}. Ancillary assumptions such as supersymmetry are not required to achieve radiative stability. The other attraction of this proposal is that since the fundamental scale of the theory is the electroweak energy scale, predictions drawn from the theory such as deviations from the $1/r^2$ law of Newtonian gravity can be experimentally tested in the near future. In the extra-dimensions scenario all of the interactions, gravity as well gauge interactions, become unified at the electroweak scale. This means that if the model is viable, particle accelerators such as the LHC, the VLHC and the NLC will be able to uncover the features of quantum gravity as well as the mechanism of electroweak symmetry breaking.

The large-extra-dimension scenario also has significant implications for processes involving strong gravitational fields, such as the bending of light near a massive object and the decay of black holes. The latter phenomenon has been described within the context of the microcanonical ensemble (see Refs. [3,4]) in four space-time dimensions. Our starting point is the idea that black holes are (excitations of) extended objects (p-branes), a gas of which satisfies the bootstrap condition. This yields a picture in which a black hole and the emitted particles are of the same nature and an improved law of black hole decay which is consistent with unitarity (energy conservation). This idea has been bolstered by recent results in fundamental string theory, where it is now accepted that extended Dp-branes are a basic ingredient [5]. States of such objects were constructed which represent black holes [6] and corroborate [7] the old idea that the area of the horizon is a measure of the quantum degeneracy of the black hole [8]. However, this latter approach works mostly for very tiny black holes and suffers from the same shortcoming that the determination of the space-time geometry during the evaporation is missing.

Embedding a black hole in a space-time of higher dimensionality would seem, from the string theory point of view, to be a natural thing to do. In models with extra spatial dimensions the four dimensional spacetime is viewed as a D3-brane embedded in a bulk spacetime of dimension $4+d$. In the most recent manifestation of string theory, matter is described by open strings whose endpoints are Dirichlet branes. Therefore in the large-extra-dimension scenario matter fields at low energy (in the present case this means at energies less than Λ_{EW}) are confined to live on the D3-brane. In string theory gravity is described by closed strings and hence it can propagate in the bulk. Black holes in $4+d$ extra dimensions have been studied in both compact [9] and infinitely extended [10] extra dimensions (see also [11] and references therein).

In Ref. [12] we studied the evaporation of black holes in space-times with extra dimensions of size L. We first obtained a description which interpolated between the expected behaviors of very large and very small black holes and then showed that a (first order) phase transition, possibly signaled by an outburst of energy, occurs in the system when the horizon shrinks below L from a larger value. The phase transition is related to both a change in the topology of the horizon and the breaking of translational symmetry along extra dimensions. In the present work we extend the results of [12] by investigating several factors which affect the decay rate of a black hole.

In Section II we review and improve on the effective potential discussed in Ref. [12]. In Section III we then analyze the evaporation of a black hole in $4+d$ dimensions both from the canonical and the microcanonical point of view and show that the luminosity of small black holes, such as primordial black holes (PBHs) which might be sources of γ-ray bursts, strongly depends on the number of space-time dimensions. Such a dependence is further enhanced by the analysis of wave propagation in Section III C where we argue that the heighth of the potential barrier which affects the grey-body factor depends on d. The phase transition and the accompanying topology change which a black hole undergoes as its horizon radius becomes of the order of the size of the extra dimensions ($R_H \sim L$) are discussed in Section IV. Finally, in Section V we summarize and discuss our results. We use units with $c = \hbar = 1$.

II. THE EFFECTIVE POTENTIAL

If large extra spatial dimensions exist in nature, deviations from Newton's law will be detected at the scale of the extra dimensions. Assuming that all of the matter described by the standard model lives on a four-dimensional D3-brane, the form of Newton's law can be obtained for a point-like mass by means of Gauss' law [1]. Denoting by r the radial distance in $4+d$ dimensions and by r_b the radial distance as measured on the D3-brane, we find for distances r much greater than the typical size of the extra dimension L a potential of the form

$$V_{(4)} = -G_N \, \frac{M}{r_b} \ , \tag{II.1}$$

where $G_N = m_p^{-2}$ is Newton's constant in four dimensions. On the other hand for $r \ll L$ the potential becomes

$$V_{(4+d)} = -G_{(4+d)} \, \frac{M}{r^{1+d}} \ , \tag{II.2}$$

with $G_{(4+d)} = M_{(4+d)}^{-2-d} = L^d \, G_N$. This implies that the huge Planck mass $m_p^2 = M_{(4+d)}^{2+d} \, L^d$ and, for sufficiently large L and d, the bulk mass scale $M_{(4+d)}$ (eventually identified with the fundamental string scale) can be as small as $1\,\text{TeV}$. Since

$$L \sim \left[1\,\text{TeV}/M_{(4+d)}\right]^{1+\frac{2}{d}} \, 10^{\frac{31}{d}-16}\,\text{mm} \ , \tag{II.3}$$

requiring that Newton's law not be violated for distances larger than 1 mm restricts $d \geq 2$ [1,13]. Further bounds are obtained by estimating the production of KK-gravitons and support higher values of d [14].

Since our purpose is to study the decay of black holes, we would like to be able to describe the metric tensor elements of a space surrounding a black hole for all values of the horizon R_H of the black hole, including $R_H \sim L$. Black holes with both very large horizon radii ($R_H \gg L$) and very small radii ($R_H \ll L$) have been extensively investigated. In the former case, the compact extra dimensions can be unwrapped and the real singularity can be regarded as spread along a (black) d-brane [15] of uniform density M/L^d, thus obtaining the Schwarzschild metric on the orthogonal D3-brane, in agreement with the weak field limit $V_{(4)}$, and an approximate "cylindrical" horizon topology $S^2 \times \mathbb{R}^d$. In the latter case a solution is known [10], for one infinite extra dimension [2], which still has the form of a black string extending all the way through the bulk AdS_5. However, this solution is unstable [16] and believed to further collapse into one point-like singularity [10]. This can be also argued from the observation that the Euclidean action of a black hole is proportional to its horizon area and is thus minimized by the spherical topology S^{2+d}. Hence, small black holes are expected to correspond to a generalization of the Schwarzschild metric to $4+d$ dimensions [17] and should be colder and (possibly much) longer lived [9].

As a black hole evaporates the topology of the horizon changes from $S^2 \times R^d$ to S^{2+d} when R_H decreases from $R_H > L$ to $R_H < L$. For small black holes a complete description would provide an explicit matching between the cylindrical metric (for $r \gg L$) and the spherical metric (for $r \ll L$). This is not a trivial detail, since the ADM mass of a spherical $4+d$ dimensional black hole is zero as seen from the D3-brane because there is no $1/r_b$ term in the large r expansion of the time-time component of the metric tensor [17]. Thus, one concludes that the four-dimensional ADM mass M of a small black hole can be determined as a function of the horizon radius R_H (which governs the evaporation) only after such a matching is provided explicitly. One can further guess that M emerges either as a *boundary effect* (*i.e.*, due to the size of the extra-dimension being bounded) or as a consequence of a non-zero tension (energy density) on the D3-brane (this latter possibility will not be considered here). Even less is known about black holes of size $R_H \sim L$, and a complete description is likely to be achieved only by solving the entire set of field equations for an evaporating black hole in $4+d$ dimensions.

Rather than attempting the formidable problem of obtaining a complete description of an evaporating black hole from the field equations, we approximate the (time and radial components of the) metric in $4+d$ dimensions as

$$g_{tt} \simeq -1 - 2\,V(r)$$
$$g_{rr} \simeq -g_{tt}^{-1}\ , \tag{II.4}$$

where

$$V(r) = -G_N\,\frac{M}{r_b}\,\Theta(r_b - L) - \sum_{n=1}^{d} \frac{C_n\,L^n\,G_N\,M\,\Theta(L-r_b)}{\left[r_b^2 + D_n\,\Theta(L-r_b)\,\sum_{i=1}^{d}\,y_i^2\right]^{(1+n)/2}}\ , \tag{II.5}$$

where Θ is the Heaviside function (or a smooth approximation of it), the y_is are cartesian coordinates in the d extra dimensions and C_n, D_n are numerical coefficients. This yields M as the ADM mass (on the D3-brane) of the black hole and the radius of the horizon is determined by

$$g_{rr}^{-1} = 0\ . \tag{II.6}$$

The above *ansatz* does not provide an exact solution of the vacuum Einstein equations, since some of the components of the corresponding Einstein tensor in $4+d$ dimensions $G_{ij} = 8\,\pi\,G_{(4+d)}\,T_{ij} \neq 0$. However, the coefficients C_n and D_n can be chosen in such a way that the "effective matter contribution" T_{ij} from the region outside the black hole horizon is small. In particular, one can require the contribution to the ADM mass to be negligible,

$$\begin{aligned} m &\equiv \int_{R_H}^{\infty} d^{4+d}x\,T^t{}_t \\ &= \frac{1}{8\,\pi\,G_{(4+d)}}\int_{R_H}^{\infty} d^{4+d}x\,G^t{}_t(\{C_n\}) \ll M\ . \end{aligned} \tag{II.7}$$

In this sense one can render the above metric a good approximation to a true black hole in $4+d$ dimensions. For instance, for $d=1$, we obtain

$$V(r) = \begin{cases} -G_N\,\dfrac{M}{r_b} & r > a\,L \gg L \\[2ex] -G_N\,\dfrac{M}{r_b} - C\,\dfrac{L\,G_N\,M}{r_b^2 + y^2} & L < b\,L < r < a\,L \\[2ex] -C\,\dfrac{L\,G_N\,M}{r_b^2+y^2} & r < b\,L\ , \end{cases} \tag{II.8}$$

where the various coefficients must satisfy

$$\frac{m}{M} = C\,\frac{a-b}{a\,b} \ll 1\ , \tag{II.9}$$

which determines the size of the region in which both $1/r$ and $1/r^2$ terms are turned on [18]. Correspondingly one finds

$$R_H(y=0) = \begin{cases} 2\,G_N\,M & M \gg M_c \\[2ex] G_N\,M\left(1+\sqrt{1+\dfrac{2\,C\,L}{G_N\,M}}\right) & M \sim M_c \\[2ex] \sqrt{2\,L\,G_N\,M} & M \ll M_c\ , \end{cases} \tag{II.10}$$

where M_c is the value of M for which the usual four-dimensional horizon radius $2\,G_N\,M = L$ [the value of M for which $R_H = L$ can be obtained only after solving Eq. (II.6), which is an algebraic equation of order $d+2$]. For $M \sim M_c$ the radius of the horizon also depends on the coordinate y and is neither cylindrically nor spherically symmetric (see Fig. 1). Analogous estimates can be obtained for $d > 1$.

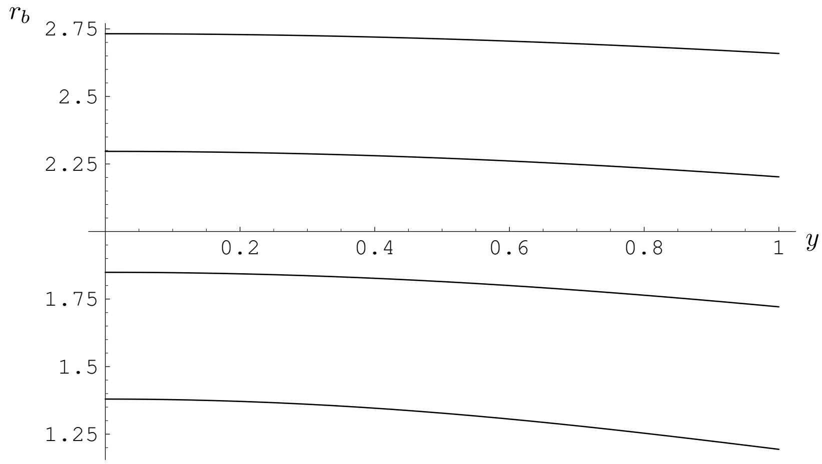

FIG. 1. Plot of the horizon for $d = 1$ and $M = M_c$, $0.8\,M_c$, $0.6\,M_c$, $0.4\,M_c$ (in units with $G = L = 1$). For $d = 0$ the corresponding horizons would be at $r_b = 2$, 1.6, 1.2, 0.8.

Finally, we note that, although the ansatz in Eq. (II.4) is not a true vacuum solution, such a solution would not be very useful for $R_H \ll L$ in any event, because once Hawking radiation is included, its backreaction on the metric at small r is likely to be significant. Therefore, it is sufficient to impose the condition that m be comparable with the energy density of Hawking's radiation integrated from $r = R_H$ to $r = \infty$, *i.e.* the amount of mass lost into Hawking particles. This quantity of course depends on the initial mass of the black hole.

III. BLACK HOLE EVAPORATION

There are several factors affecting the rate of black hole evaporation due to Hawking radiation. We begin with a review of the expression for the decay as obtained from the canonical ensemble and then discuss the modifications to this expression which arise when the more basic microcanonical ensemble is used. Finally, we estimate the grey-body factor by studying the Klein-Gordan equation.

A. The canonical picture

When the size of the black hole is large compared to the extra dimensions ($R_H \gg L$), $r \simeq r_b$ and the metric (II.4) is approximately cylindrically symmetric (along the extra dimensions). Further, when L is set to 0, $m = 0$, and Eq. (II.6) coincides with the usual four dimensional Schwarzschild radius

$$R_H \simeq 2\,\ell_p\,\frac{M}{m_p}\ . \tag{III.1}$$

Correspondingly the inverse Hawking temperature ($\beta_H = T_H^{-1}$) and Euclidean action are [19,8]

$$\beta_H^{>} \simeq 8\,\pi\,\ell_p\,\frac{M}{m_p} \tag{III.2}$$

$$S_E^{>} \simeq 4\,\pi\,\left(\frac{M}{m_p}\right)^2 \simeq \frac{\mathcal{A}_{(4)}}{4\,\ell_p^2}\ , \tag{III.3}$$

where $\mathcal{A}_{(D)}$ is the area of the horizon in D space-time dimensions and the condition $R_H \gg L$ translates into

$$M \gg m_p\,\frac{L}{\ell_p} \equiv M_c \sim \left(\frac{L}{1\,\text{mm}}\right)\,10^{27}\,\text{g}\ . \tag{III.4}$$

The fact that the extra dimensions do not play any significant role at this stage is further confirmed by $T_H^{>} \ll 2\,\pi/L$. Since $2\,\pi/L \equiv m^{(1)}$ is the mass of the lightest Kaluza-Klein(KK) mode, no KK particles can be produced.

The energy loss per unit time for an evaporating black hole is given by

$$\frac{dM}{dt} = -F^{>}_{(4+d)} \simeq -F^{>}_{(4)} \ , \tag{III.5}$$

where $F_{(D)}$ is the total luminosity as measured in D space-time dimensions. Since microcanonical corrections have been shown to become significant only for $M \sim m_p$ (in four space-time dimensions [4]), one can further approximate the luminosity by employing the (simpler) canonical expression [19]

$$\begin{aligned} F_{(D)} &\simeq \mathcal{A}_{(D)} \sum_s \int_0^\infty \frac{\Gamma_s(\omega)\,\omega^{D-1}}{e^{\beta_H\,\omega} \mp 1}\, d\omega \\ &= \mathcal{A}_{(D)}\, N_{(D)}\, (T_H)^D \ , \end{aligned} \tag{III.6}$$

where Γ is the grey-body factor and N is a coefficient which depends upon the number of available particle species s with energy smaller than T_H and is also affected by the value of Γ. For $D = 4$ and $\beta_H = \beta_H^{>}$ one obtains the well known result [19]

$$F^{>}_{(4)} \sim \frac{N_{(4)}}{\ell_p^2} \left(\frac{m_p}{M}\right)^2 \ . \tag{III.7}$$

For $R_H \ll L$ the parameters in the potential (II.5) can be chosen to minimize the contribution of m to the ADM mass and this is generally obtained by switching off all terms except that going as $1/r^{1+d}$ for $r < L$. Eq. (II.6) then leads to

$$R_H \simeq \left(2\, L^d\, G_N\, M\right)^{1/1+d} \ , \tag{III.8}$$

and the consistency conditions $\ell_p \ll R_H \ll L$ hold for

$$m_p \left(\frac{\ell_p}{L}\right)^{\frac{d}{2+d}} \ll M \ll M_c \ . \tag{III.9}$$

Since we have assumed that the spherical symmetry extends to $4+d$ dimensions, one obtains [9]

$$\beta_H^{<} \sim L \left(\frac{M}{M_c}\right)^{\frac{1}{1+d}} \tag{III.10}$$

$$S_E^{<} \sim \left(\frac{L}{\ell_p}\right)^2 \left(\frac{M}{M_c}\right)^{\frac{2+d}{1+d}} \sim \frac{A_{(4+d)}}{\ell_p^2\, L^d} \ , \tag{III.11}$$

which reduce back to (III.2) and (III.3) if one pushes down $L \to \ell_p$ ($M_c \to m_p$). For $M \ll M_c$, the temperature $T_H^{<}$ is sufficient to excite KK modes, although it is lower than that of a four-dimensional black hole of equal mass. Correspondingly, the Euclidean action $S_E^{<}(M) \geq S_E^{>}(M)$, yielding a smaller tunnelling probability [19,3]

$$P \sim \exp\left(-S_E\right) \ , \tag{III.12}$$

that is, a smaller probability for the Hawking particles to be created in the $4+d$ dimensional scenario.

Although ordinary matter is confined on the D3-brane, a black hole can emit particles via Hawking's process into all of the $3+d$ spatial directions of the bulk. For $R_H \ll L$ the evaporation of a black hole can be obtained from Eq. (III.6) with $D = 4+d$ and $T_H = T_H^{<}$,

$$\frac{dM}{dt} = -F^{<}_{(4+d)} \sim \frac{N_{(4+d)}}{L^2} \left(\frac{M_c}{M}\right)^{\frac{2}{1+d}} \ . \tag{III.13}$$

The luminosity (III.6) can be written as

$$F_{(D)} = \frac{N_{(D)}}{R_H^2} \ . \tag{III.14}$$

Thus, a comparison of the radius of the (apparent) horizon (III.8) with the analogous quantity in Eq. (III.1) shows that the luminosity of a black hole of given ADM mass $M < M_c$ is much smaller in $4+d$ dimensions than it would be with no extra dimensions. This assertion might have to be qualified if the $N_{(D)}$ term in the luminosity were greater in $4+d$ dimensions than the one in four dimensions. We show below that this is not the case.

Conservation of energy in $4+d$ dimensions requires that

$$F_{(4+d)} = F_{(4)} + F_{KK} \tag{III.15}$$

and since $F_{(D)} \sim N_{(D)}$ [Eq. (III.6)] and the number of degrees of freedom of KK gravitons is much less than the number of standard model particles ($N_{KK} \ll N_{(4)}$), the energy emitted into the KK modes must be a small fraction of the total luminosity (similar results were obtained in Ref. [11])

$$\frac{F_{KK}}{F_{(4+d)}} \sim \frac{N_{KK}}{N_{(4)} + N_{KK}} \ll 1 \; . \tag{III.16}$$

Standard model particles on the D3-brane with high enough energy (*e.g.*, larger than Λ_{EW}) might be able to overcome the confining mechanism. In this case the bulk standard model fields should be included among the KK modes and N_{KK} should be considered as an increasing function of the temperature. In this scenario the ratio $F_{KK}/F_{(4+d)}$ would eventually reach unity when $T_H^< \gg \Lambda_{EW}$.

B. The Microcanonical Picture

The canonical ensemble description used above is in principle incorrect because a black hole in asymptotically flat space-time cannot be in thermal equilibrium with its Hawking radiation. However for large black holes this description is a very good approximation to the true picture. The correct statistical mechanical description of black holes utilizes the microcanonical ensemble [3,4].

For $R_H \gg L$ the topology of the horizon is "cylindrical", and the Euclidean action is

$$S_E^> \simeq 4\,\pi \left(\frac{M}{m_p}\right)^2 \; . \tag{III.17}$$

The number density in the microcanonical enesemble for this case is

$$n^>(\omega) = \sum_{l=1}^{[[M/\omega]]} \frac{\exp\left[4\,\pi\,(M - l\,\omega)^2/m_p^2\right]}{\exp(4\,\pi\,M^2/m_p^2)} \; , \tag{III.18}$$

where $[[X]]$ denotes the integer part of X. In the limit $M \to \infty$, $n^>$ is equal to the canonical ensemble number density used in Eq. (III.6). The decay rate for a black hole in $4+d$ dimensions is given by

$$\frac{dM}{dt} = \mathcal{A}_{(4+d)} \int_0^\infty n\,(\omega)\,\Gamma(\omega)\,\omega^{d+3}\,d\omega \; . \tag{III.19}$$

Making the substitutions $n = n^>$, $x = M - l\,\omega$ and assuming that $\Gamma \simeq 1$, the decay rate in four dimensions can be written as

$$\frac{dM}{dt} = \mathcal{A}_{(4)}\,I \sum_{l=1}^{\infty} \frac{1}{l^4} = \frac{\pi^4}{90}\,\mathcal{A}_{(4)}\,I \; , \tag{III.20}$$

where

$$I = \int_0^M \frac{(M-x)^3\,\exp\left(4\,\pi\,x^2/m_p^2\right)\,dx}{\exp\left(4\,\pi\,M^2/m_p^2\right)} \; . \tag{III.21}$$

A plot of the decay rate as a function of the mass is shown in Fig. 2. However, we remark that, in the presence of extra dimensions, this description holds only for $M > M_c$ ($R_H > L$), therefore the plot in Fig. 2 is probably not accurate for small masses. It should in fact be considered only in the region $M/m_p > L/\ell_p$ where there is no essential difference between the microcanonical and the canonical luminosities (III.7).

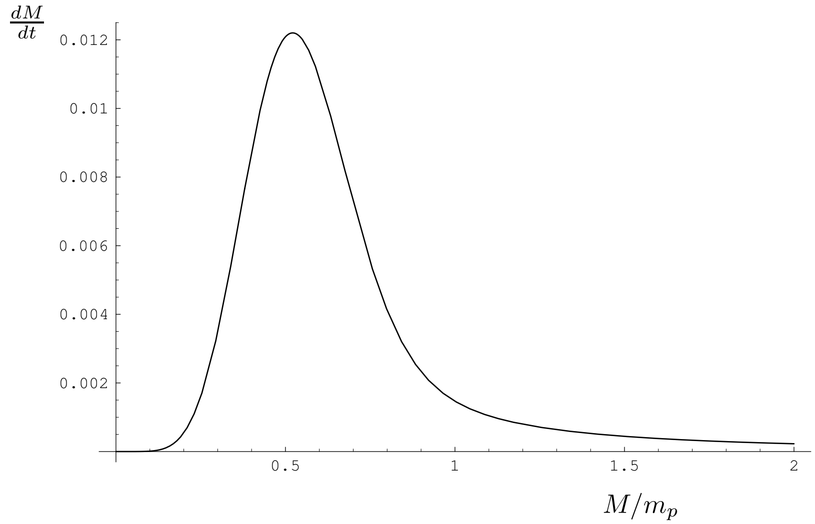

FIG. 2. Decay rate for a large ($R_H \gg L$) black hole.

For $R_H < L$ the Euclidean action is

$$S_E^{<} = 4\,\pi \left(\frac{L}{\ell_p}\right)^2 \left(\frac{M}{M_c}\right)^{(d+2)/(d+1)} , \tag{III.22}$$

where $M_c = m_p\, L/\ell_p$. The number density for horizon radii of this size is

$$n^{<}(\omega) = \sum_{l=1}^{[[M/\omega]]} \frac{\exp\left[4\,\pi\,\left(\frac{M-l\,\omega}{M_c}\right)^{\frac{d+2}{d+1}}\,\left(\frac{L}{\ell_p}\right)^2\right]}{\exp\left[4\,\pi\,\left(\frac{M}{M_c}\right)^{\frac{d+2}{d+1}}\,\left(\frac{L}{\ell_p}\right)^2\right]} . \tag{III.23}$$

The decay rate corresponding to this number density as calculated from Eq. (III.19) is exhibited for $d = 0, \ldots, 4$ in Fig. 3. In this plot, we have matched the microcanonical decay rates for $M < M_c$ with the canonical rate given in Eq. (III.7) for $M > M_c$ by multiplying the former by a suitable constant in order to show that there is a peak in the luminosity around M_c. In fact, the curves show clearly that the decay rates for all cases with $d > 1$ have a maximum at $M \simeq M_c$ and black holes with masses less than M_c thus have a significantly slower evaporation rate which results in a longer lifetime (see Fig. 4). The case $d = 1$ appears slightly different, but this value of d must be rejected because it leads to contradictions with experimental tests of Newton's law [1,13].

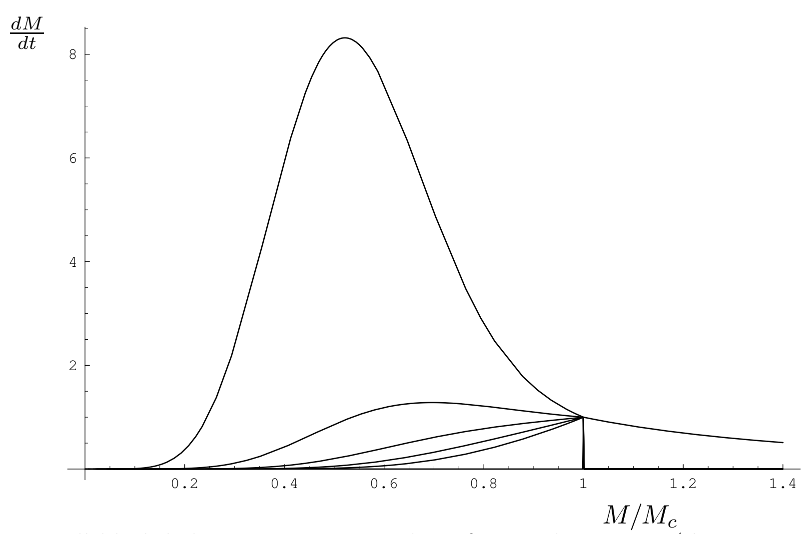

FIG. 3. Decay rate for a small black hole in increasing number of extra dimensions ($d = 0$ uppermost curve, $d = 4$ lowest curve). Vertical units are arbitrary.

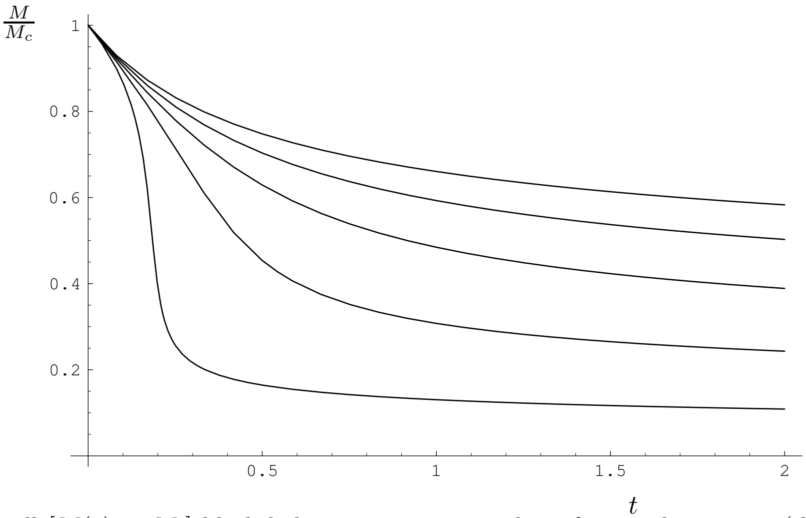

FIG. 4. History of a small $[M(0) \sim M_c]$ black hole in increasing number of extra dimensions ($d = 0$ lowest curve, $d = 4$ uppermost curve). Horizontal units are arbitrary.

We wish to make two remarks about the above estimates: firstly, we expect the precise shapes of the decay rates in Fig. 3 at $M \sim M_c$ depend on the details of the phase transition which we are able to describe only qualitatively (see Section IV). As a consequence, the curves in Fig. 4 are not totally reliable around $t = 0$ ($M \sim M_c$), but can be used to describe black holes which are produced with $M(0)$ significantly less than M_c; secondly, we have always set $\Gamma \sim 1$, but the grey-body factor actually depends strongly on the number of extra dimensions d causing a further suppression of the decay rate, as we show in Section III C.

A reduced decay rate for black holes in large extra dimensions has several interesting implications for cosmological phenomena involving PBHs. For $d = 2$ and $M_{(6)} \sim 1\,\text{TeV}$, the size of the extra dimension is $L \sim 1\,\text{mm}$ [from Eq. (II.3)] and $M_c \sim 10^{27}\,\text{g}$ [from Eq. (III.4)]. Since the initial mass of an astrophysical black hole is known to exceed at least a few solar masses ($M_0 > 10^7\, M_c$) [23], its $T_H^{>}$ is cooler than the cosmic background radiation and it cannot decay to M_c in any reasonable amount of time. However, PBHs (see, *e.g.*, Refs. [24]) could have been produced with an initial mass $M_0 \ll M_c$, so that $R_H \ll L$ from the very beginning and, according to Fig. (3) the lifetimes of such black holes would be many orders of magnitude longer than those living in four spacetime dimensions. For instance, a PBH with $M_0 \sim 10^{15}\,\text{g}$, which has a life-time comparable with the age of the Universe according to Eq. (III.7) (the canonical ensemble is a good approximation in this mass range for $d = 0$), in this new scenario would decay in $\tau_{(6)} \sim 10^{16}$ times the age of the Universe. On the other hand, if the PBH had been produced with $R_H > L$, we can speculate that, after it shrank down to near L according to Eq. (III.7), either the evaporation continued and its mass eventually entered into the regime of very slow decay, or the evaporation stopped (because of the backreaction) at $M \sim M_c$. The latter option is even more dramatic, since it would leave remnants of mass as large as M_c (to be compared with possible remnants of Planck size [24] as follows from microcanonical estimates in four dimensions [4]).

One can therefore highlight a list of topics involving PBHs upon which models with extra dimensions might have a bearing [9]:

1. PBHs could be the source of γ-ray bursts and cosmic rays only if their initial mass was such that $M \ll 10^5\,\text{g}$ [25] or $M \sim M_c$ at the time of the emission; the allowed density of PBHs which fits into bounds obtained from current observations could then be significantly changed, with consequences on

2. models of the early Universe,

3. dark matter (in the form of KK particles [1]) and

4. density and size of microlensing sources [26].

C. Angular Momentum Barrier to Decay

A scalar wave in $4 + d$ dimensions satisfies the equation

$$\Box\, \Phi = 0 \;, \tag{III.24}$$

where the D'Alembertian is given by

$$\Box = \frac{1}{\sqrt{-g}}\,\partial_\mu\,\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\right)\ , \tag{III.25}$$

with (in spherical coordinates for which $\theta_1 = \theta$ and $\theta_2 = \phi$)

$$\sqrt{-g} = r^{2+d}\,\prod_{i=1}^{d+2}(\sin\theta_i)^{d+2-i} \tag{III.26}$$

For $L \ll R_H < r$ we can neglect the extra dimensions and simply take the standard Schwarzschild line element on the brane,

$$ds^2 \simeq -\Delta_{(4)}\,dt^2 + \frac{dr^2}{\Delta_{(4)}} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\ , \tag{III.27}$$

where

$$\Delta_{(D)} = 1 - \left(\frac{R_H}{r}\right)^{D-3}\ . \tag{III.28}$$

For $R_H < r \ll L$ one can analogously consider a spherically symmetric black hole in $4+d$ dimensions [17]. However, in order to take into account the fact that d spatial dimensions have size L, we shall instead use the following form

$$ds^2 \simeq -\Delta_{(4+d)}\,dt^2 + \frac{dr^2}{\Delta_{(4+d)}} + r^2\ \left(d\theta^2 + \sin^2\theta\,d\phi^2\right) + r^2\,\sum_{i=1}^{d} d\phi_i^2\ , \tag{III.29}$$

where $R_H < r < L$ is the $4+d$ dimensional areal coordinate and $dx^i \simeq r\,d\phi_i$ are cartesian coordinates in the extra dimensions.

We assume the scalar field Φ can be factorized according to

$$\Phi = e^{i\,\omega\,t}\,R(r)\,S(\theta)\,e^{i\,m\,\phi}\,e^{i\,\sum\,n_i\,\phi_i}\ , \tag{III.30}$$

with n_i positive integers, so that Φ satisfies periodic boundary conditions at the edges of the bulk ($y_i = \pm L/2$). The radial equation obtained from Eq. (III.24) for the metric (III.29) then becomes ($\Delta \equiv \Delta_{(4+d)}$)

$$\left[-\frac{\Delta}{r^{2+d}}\,\frac{d}{dr}\,\left(r^{2+d}\,\Delta\,\frac{d}{dr}\right) + \omega^2 - \frac{\Delta}{r^2}\,A\right]\,R = 0 \tag{III.31}$$

and the angular equation is

$$\left[\frac{1}{\sin\theta}\,\frac{\partial}{\partial\theta}\,\left(\sin\theta\,\frac{\partial}{\partial\theta}\right) - \frac{m^2}{\sin^2\theta}\right]\,S = \lambda\,S\ , \tag{III.32}$$

where $A = \lambda + \sum n_i^2$, the separation constant $\lambda = l\,(l+1)$ and $S = Y_l^m$ is a standard spherical harmonic.

The radial equation can be further simplified by defining a tortoise coordinate

$$dr_* \equiv \frac{dr}{\Delta}\ , \tag{III.33}$$

and introducing a rescaled radial function,

$$W \equiv r^{1+d/2}\,R\ , \tag{III.34}$$

which then satisfies a Schrödinger-like equation

$$\left[-\frac{d^2}{dr_*^2} + V\right]\,W = \omega^2\,W\ , \tag{III.35}$$

where the potential $V = V_1 + V_2$ is given by

$$V_1 = \left[(1+d)\,\left(1+\frac{d}{2}\right) + A\right]\,\frac{\Delta}{r^2}$$

$$V_2 = -\,\left(1+\frac{d}{2}\right)^2\,\frac{\Delta^2}{r^2}\;. \tag{III.36}$$

The contribution V_2 vanishes sufficiently fast near R_H and is negligible there. The potential V_1 generates a barrier located outside the horizon which suppresses the grey-body factor for all modes of the scalar field including those with zero angular momentum [27]. The effect for $d = 0$ is mild, however for $d > 0$ the barrier increases, significantly reducing the black hole decay rate. From the plot of the potential in Fig. 5 one can estimate the (frequency dependent) suppression factors with respect to the purely four-dimensional case (for which $\Gamma \sim 1$) by making use of the W.K.B. approximation for the transmission probability

$$\Gamma(\omega) \sim \exp\left(-2\int dr\,\sqrt{|V-\omega|}\right)\;, \tag{III.37}$$

where the integral must be performed between the two values of r at which $V = \omega$ (for ω smaller than the maximum of V). For the typical frequency $\omega \sim T_H^<$ (~ 1 in the units of Fig. 5) one obtains

$$\begin{aligned}
&\Gamma_{d=1} \sim 1\\
&\Gamma_{d=2} \sim 0.73\\
&\Gamma_{d=3} \sim 0.22\\
&\Gamma_{d=4} \sim 0.05\;.
\end{aligned} \tag{III.38}$$

One then observes that, for frequency $\omega < T_H^<$ the transmission probability is smaller, while for quanta of frequency $\omega \gg T_H^<$ the probability of creation is actually negligible. We remark that the curves shown in Fig. 5 were obtained by setting $A = 0$ (*i.e.*, $l = n_i = 0$) and, since the n_is cannot be zero, the curves represent lower bounds on the heighths of the curves and the values in Eq. (III.38) represent upper bounds on the grey-body factors. This barrier effect taken together with the decay rate reduction for $d > 1$ (again, we recall that the case $d = 1$ is ruled out) as shown in Fig. 3 would seem to suggest that black holes with horizon radii on the order of the size of the extra dimensions or less decay very little if at all.

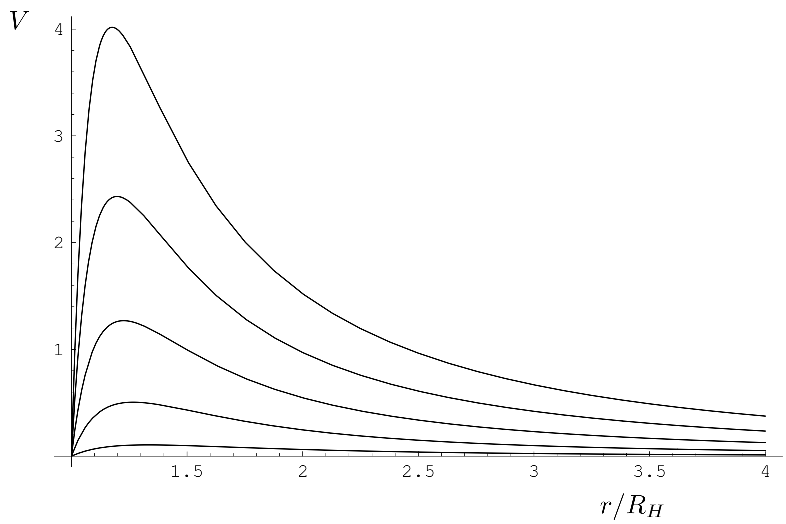

FIG. 5. Potential $V_1 + V_2$ for different numbers of extra dimensions ($d = 4$ uppermost curve, $d = 0$ lowest curve).

IV. HORIZON TOPOLOGY CHANGE AND PHASE TRANSITION

In the foregoing analysis horizon radii on the order of the size of the extra dimensions ($R_H \sim L$) have not been considered in great detail. For example, the expressions for β_H and S_E in this region are discontinuous because they contain numerical coefficients which depend on the topology and number of the extra dimensions. For instance,

$$S_E^<(M_c) \simeq \frac{S_{(4+d)}}{S_{(4)}}\, S_E^>(M_c)\ . \tag{IV.1}$$

A complete description of this process likely requires solving the full quantum backreaction problem in the bulk when $M \sim M_c$, but we can get further physical insight by appealing to the point of view of the statistical mechanics. Although the use of the canonical ensemble is incorrect, it still leads to results in fairly good agreement with the microcanonical description for black holes of mass greater than m_p in four dimensions and greater than M_c in $4+d$ dimensions. We can then introduce an effective partition function for the black hole and its Hawking radiation as the Laplace transform of the microcanonical density of states but with a cut-off Λ which makes the partition function integral finite,

$$Z_\beta \sim \int^\Lambda dM\, e^{-\beta\, M}\, e^{S_E(M)}\ . \tag{IV.2}$$

It then follows from Eq. (IV.1) that there is a discontinuity in the first derivative of the free energy $\beta\, F \equiv -\ln Z$ at

$$\beta_c \sim L \sim \frac{1}{m^{(1)}}\ , \tag{IV.3}$$

that is a first order phase transition (see Fig. 6): In the cold phase, $T_H < \beta_c^{-1}$, the system appears "condensed" into the lower four dimensional 3-brane and translational invariance in the d extra directions is broken. For $T_H > \beta_c^{-1}$ translational invariance is restored, and the system starts spreading over all bulk space, with brane vibrations playing the role of Nambu-Goldstone bosons which give mass to the KK modes [1]. In this approximation the specific heats in the cold and hot phases are given by

$$C_V^> \sim -\left(\frac{M}{m_p}\right)^2 \tag{IV.4}$$

$$C_V^< \sim -L\, M \left(\frac{M}{M_c}\right)^{\frac{1}{1+d}}\ , \tag{IV.5}$$

Since $C_V^<$ is negative (as is $C_V^>$), one is eventually forced to use the microcanonical description when the temperature is too high [4], as we have done in Section III B.

We end this Section by noting that the above analysis does not determine the typical times of such a transition. Using the results of the previous Section, one could argue that the transition takes a very long time to occur (if at all), since the luminosity of black holes is strongly damped for $M < M_c$. Although an outburst of energy at $M \sim M_c$ cannot be excluded (and is actually favored according to the microcanonical rate as displayed in Fig. 3), it seems implausible that the black hole can then complete the transition and reach very small masses in a reasonable amount of time.

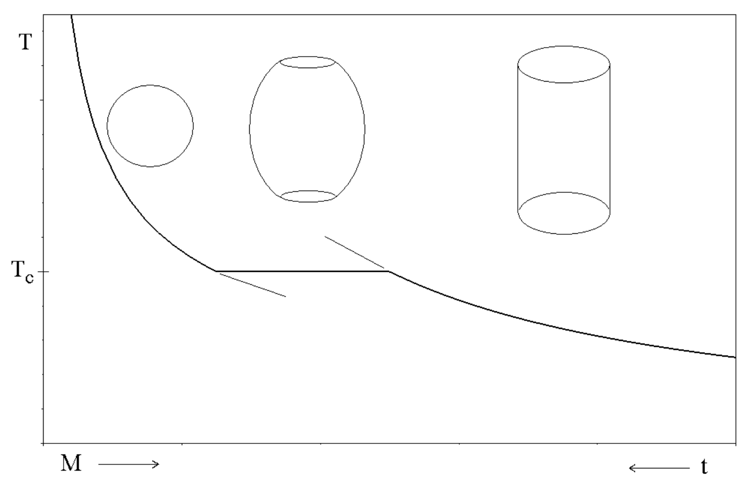

FIG. 6. Sketch of the phase transition described in the text. Note that the shape of the horizon along the extra dimension(s) for $T \sim T_c$ is shown in Fig. 1 (for $d = 1$).

V. CONCLUSIONS

While the effective potential of Eq. (II.4) is not a vacuum solution, it does have the desirable property of providing only a small contribution to the ADM mass if the adjustable constants in the potential are properly chosen. It also contains a parameter which is readily identifiable as the mass of the black hole for both $R_H > L$ and $R_H < L$.

The analysis of the black hole decay rate in both the canonical and microcanonical pictures shows that the presence of large extra dimensions would slow the decay rate, and that the decay rate decreases with increasing number of extra dimensions. The source of this decrease in decay rate is the dependence of the number density and the horizon area on the number of extra dimensions. In addition to these effects the size of the angular momentum barrier increases with the number of extra dimensions, further decreasing the decay rate. This result suggests that, even without taking backreaction effects into account, black holes with horizon radii less than the size of the extra dimensions are quasi-stable. Thus, even small primordial black holes (masses less than $M_0 \sim 10^{15}$ gm) could have lifetimes comparable to or even much longer than the age of the present universe. The detection of PBHs with masses less than M_0 would be evidence for the existence of large extra dimensions, and the values of the PBH masses would set limits on the number of extra dimensions.

A statistical mechanical analysis of a black hole whose horizon radius R_H is approximately equal to the size L of the extra dimensions shows that as R_H shrinks below L, a phase transition occurs. This event is accompanied by a change in the horizon topology from "cylindrical" ($S^2 \times R$) to "spherical" (S^3). An interesting extension of the present work would be to see if the phase transition can be described in terms of the topology change.

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